AP Calculus BC

WS 81 - Ratio & Root Test

1) $\sum \frac{3^k}{k!} = S$

$\lim_{k\to\infty} \left| \frac{3^{k+1}}{(k+1)!} \cdot \frac{k!}{3^k} \right|$

$\lim_{k\to\infty} \left| \frac{3}{k+1} \right| = 0 < 1$

S converges by Ratio Test

2) $\sum \frac{4^k}{k^2} = S$

(* Also $n^{th}$ term test)

$\lim_{k\to\infty} \left| \frac{4^{k+1}}{(k+1)^2} \cdot \frac{k^2}{4^k} \right|$

$\lim_{k\to\infty} \left| \frac{4k^2}{(k+1)^2} \right| = 4 > 0$

S diverges by Ratio Test

3) $\sum \frac{k!}{k^3} = S$

$\lim_{k\to\infty} \left| \frac{(k+1)!}{(k+1)^3} \cdot \frac{k^3}{k!} \right|$

$\lim_{k\to\infty} \left| \frac{k+1}{(k+1)^3} \cdot k^3 \right|$

$\lim_{k\to\infty} \left| \frac{k^3}{(k+1)^2} \right| = \infty$

S diverges by Ratio Test

4) $\sum \left( \frac{4n-5}{2n+1} \right)^n = S$

$\lim_{n\to\infty} \sqrt[n]{\left( \frac{4n-5}{2n+1} \right)^n}$

$\lim_{n\to\infty} \left( \frac{4n-5}{2n+1} \right) = 2 > 1$

S diverges by Root

5) $\sum \frac{k}{5^k} = S$

$\lim_{k\to\infty} \sqrt[k]{\frac{k}{5^k}} = \frac{1}{5} < 1$

S converges by Root Test

6) $\sum \frac{2^k}{k^3} = S$

$\lim_{k\to\infty} \frac{2^k}{k^3} = \infty$

S diverges by $n^{th}$ term test

7) $\sum \frac{7^k}{k!} = S$

$\lim_{k\to\infty} \left| \frac{7^{k+1}}{(k+1)!} \cdot \frac{k!}{7^k} \right|$

$\lim_{k\to\infty} \frac{7}{k+1} = 0 < 1$

S converges by Ratio Test

8) $\sum \frac{k^2}{5^k} = S$

$\lim_{k\to\infty} \sqrt[k]{\frac{k^2}{5^k}} = \frac{1}{5} < 1$

S converges by Root test

9) $\sum k^{50} e^{-k} = \sum \frac{k^{50}}{e^k} = S$

$\lim_{k\to\infty} \sqrt[k]{\frac{k^{50}}{e^k}}$

$\lim_{k\to\infty} \frac{(\sqrt[k]{k})^{50}}{\sqrt[k]{e^k}} = \frac{1}{e} < 1$

S converges by Root Test

10) $\sum \frac{\cos n\pi}{\sqrt{n}} = S$

$b_n = \frac{1}{n}$ $\quad \sum \frac{1}{n}$ diverges by p-series

$\lim_{n\to\infty} \left[ \frac{\cos n\pi}{\sqrt{n}} \cdot n \right] = \infty$

S diverges by LCT

11) $\sum \frac{1}{2\sqrt{n}+\sqrt[3]{n}} = S$

$b_n = \frac{1}{n}$ $\quad \sum \frac{1}{n}$ diverges by $p$-series

$$\lim_{n\to\infty} \left[\frac{1}{2\sqrt{n}+\sqrt[3]{n}} \cdot n\right] = \infty$$

$S$ diverges by LCT

13) $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)} = S$

$b_n = \frac{1}{n}$ $\quad \sum \frac{1}{n}$ diverges by $p$-series

$$\lim_{n\to\infty} \left[\frac{1}{\ln(\ln n)} \cdot n\right] = \infty$$

$S$ diverges by LCT

12) $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^3} = S$

$b_n = \frac{1}{n^2}$ $\quad \sum \frac{1}{n^2}$ converges by $p$-series

$$\lim_{n\to\infty} \left[\frac{(\ln n)^3}{n^3} \cdot n^2\right]$$

$$\lim_{n\to\infty} \left[\frac{(\ln n)^3}{n}\right] = 0$$

$S$ converges by LCT

14) $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^{3/2}} = S$

$b_n = \frac{1}{n^{5/4}}$ $\quad \sum \frac{1}{n^{5/4}}$ converges by $p$-series

$$\lim_{n\to\infty} \left| \frac{(\ln n)^3}{n^{3/2}} \cdot n^{5/4} \right|$$

$$\lim_{n\to\infty} \left| \frac{(\ln n)^3}{n^{1/4}} \right| = 0$$

$S$ converges by LCT